MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

EMT1016 – ENGINEERING MATHEMATICS 1

(All Sections / Groups)

3 MARCH 2020

2:30 P.M. – 4:30 P.M.

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **6 pages** (including cover page) with **4** Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) Consider the function $f(x)=\begin{cases} x^2 & x>1, \\ -2x+a & x\le 1. \end{cases}$

Find the value of constant a such that the function $f(x)$ is continuous at $x=1$. Explain your answer clearly. [6 marks]

(b) Find $\frac{dy}{dx}$ if $y\cos(x)=x^2y^3$. [6 marks]

(c) Find the absolute maximum and absolute minimum of the function $f(x)=\frac{4}{3}x^3-9x+5$ on the interval $[0,2]$. [6 marks]

(d) Evaluate the integral

$$\int \frac{1}{(1+x)(1-x)}dx.$$ [7 marks]

Question 2

(a) The Maclaurin series for $\sin(x)$ is given as follows

$$\sin(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\cdots, \text{ for all real } x.$$

Hence, or otherwise, find the Maclaurin series for the following functions

(i) $x\sin(2x)$.

(ii) $\cos(x)$.

[12 marks]

(b) (i) Given $\frac{2i}{\sqrt{2}+\sqrt{2}\,i}=a+bi=re^{i\theta}$. Find the values of a, b, r and θ.

Hence, or otherwise, find $\left(\frac{2i}{\sqrt{2}+\sqrt{2}\,i}\right)^4$. [6 marks]

(ii) Find all the three complex roots of the equation $z^3=1+i$. [7 marks]

Continued...

Question 3

(a) A continuous random variable X has probability density function

$$f(x)=\begin{cases} Ax(2-x), & 0\le x\le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $k=3/2$. [3 marks]

(ii) Find the mean and variance of X. [7 marks]

(b) Based on historical statistics, it is determined that, on average, a junction sees 9 traffic accidents per month. Find the probability that for any given month at this junction

(i) exactly 7 accidents will occur. [3 marks]

(ii) at least 4 accidents will occur. [4 marks]

(c) The average lifetime of a microchip-making machine is 15 years, with standard deviation of 2 years. A machine is considered readily marketable if its lifetime is more than 12 years. Assume that the lifetime of these machines follow approximately a normal distribution, find the probability that a random sample of such machine is readily marketable. [8 marks]

Question 4

Consider the function $f(x)=x^2$ defined on interval $0<x\le 2$. Let $g(x)$ be the periodical extension of $f(x)$.

(a) Sketch the graph of $g(x)$ on the interval $-6<x\le 6$

(i) if $g(x)$ is an odd function of period 4. [6 marks]

(ii) if $g(x)$ is an even function of period 4. [6 marks]

(b) Obtain the Fourier Series for the $g(x)$ in (a)(i), when $g(x)$ is an odd function of period 4. [13 marks]

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APPENDIX A:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x;n,p)$	
P.m.f.	$P(X=x)={}^{n}C_{x}\,p^{x}q^{n-x}$, for $x=0,1,...,n$ and where $q=1-p$.
Mean	$E[X]=np$
Variance	$Var(X)=npq$
Hypergeometric Distribution, $X \sim h(x;N,n,k)$	
P.m.f.	$P(X=x)=\dfrac{{}^{k}C_{x}\times{}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x=0,1,...\min(n,k)$.
Mean	$E[X]=\dfrac{nk}{N}$
Variance	$Var(X)=\dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1-\dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x,\lambda)$	
P.m.f.	$P(X=x)=\dfrac{\lambda^{x}e^{-\lambda}}{x!}$, for $x=0,1,....$
Mean	$E[X]=\lambda$
Variance	$Var(X)=\lambda$

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APPENDIX B:

Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$\text{Var}(X) = \dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z = \frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

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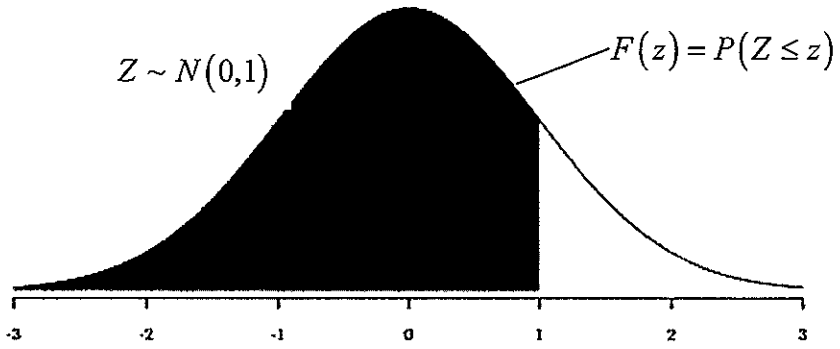
APPENDIX C:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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